But

$$\theta_{21} = \frac{f_1}{f_2}\,\theta_{22}.$$

Therefore

$$180° - \theta = \frac{f_1}{f_2}\,(180° + \theta)$$

or

$$\theta = \frac{f_2 - f_1}{f_2 + f_1}\,180°.$$

This relationship is a function only of the ratio of band edge frequencies. The characteristic impedance of the series element is

$$Z_{02} = \frac{41.4}{\tan \theta_{22}} = \frac{41.4}{\tan \theta}.$$

This quantity also is a function of the ratio of band-edge frequencies, independent of center frequency. The constant is determined by the choice of gain at center frequency and at band edges. For example, octave bandwidth at 11-db gain corresponds to 23.9 Ω.

ACKNOWLEDGMENT

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Microwave Breakdown Near a Hot Surface*

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Summary—Microwave breakdown near a hot surface in a waveguide system was studied to determine its dependence upon the thickness of the adjacent film of hot gas and its associated temperatures. The effect of the variation of the film thickness with the flow rate of the bulk of the gas was of particular interest. To carry out the theoretical analysis, a more general breakdown equation was derived to account for the temperature gradients. Experimental results supporting the theory also are presented.

The study shows that, although the breakdown threshold of a waveguide system is lowered by the presence of a hot surface, a sufficiently rapid flow of the bulk gas tends to restore the threshold as a result of the reduction in the thickness of the film of hot gas. This effect occurs in addition to that reduction resulting from cooling the surface.

INTRODUCTION

ONE OF THE PROBLEMS associated with maintaining the high power capabilities of microwave transmission lines and components is the reduction of the breakdown threshold resulting from localized heating. Breakdown may be initiated in the region of heated gas at field strengths much below that required for breakdown of the main volume, since the field strength at which ionization begins to occur is inversely proportional to the absolute value of the temperature of the gas in a constant pressure system. In the presence of localized heating, temperature gradients are established in the gas adjacent to the hot surface which in turn leads to nonequilibrium conditions characterized by convection currents. The motivation for this study was the realization that the layer of hot gas may consist of only a thin film whose thickness can be controlled by the velocity of the gas flow across the surface.[1] Thus, breakdown in the film can also be controlled.

Rapid gas flow may also contribute to an increased rate of electron loss from the region of ionization with the consequence that the breakdown threshold is raised. This effect has been reported at low pressures[2] (10 mm Hg); however, at the atmospheric pressure used in the current work, the effect was absent.

The problem of microwave breakdown under uniform conditions[3] or nonuniform conditions of electric field have been investigated over the last few years.[4,5] Practical considerations of breakdown problems in

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[1] W. H. Giedt, "Principles of Engineering Heat Transfer," D van Nostrand Co., Inc., New York, N. Y.; 1957.

[2] J. G. Skinner and J. J. Brady, "Effect of gas flow on the microwave dielectric breakdown of oxygen," *J. Appl. Phys.*, vol. 34, pp. 975–978; April, 1963.

[3] L. Gould and L. W. Roberts, "Breakdown of air at microwave frequencies," *J. Appl. Phys.*, vol. 27, pp. 1162–1170; October, 1956.

[4] M. A. Herlin and S. C. Brown, "Electrical breakdown of a gas between coaxial cylinders," *Phys. Rev.*, vol. 74, pp. 910–913; October, 1948.

[5] P. M. Platzman and E. Huber-Solt, "Microwave breakdown in nonuniform electric fields," *Phys. Rev.*, vol. 119, pp. 1143–1149; August, 1960.

microwave systems have also been discussed.[6] The purpose of this work is to extend the theoretical and experimental studies of microwave breakdown to include the case of nonuniform conditions in gas density.

Breakdown Theory

The parameter in breakdown which determines the average energy of the electrons is the ratio of electric field to pressure E_e/p where E_e is an equivalent value determined by the frequency of operation[3] and pressure enters as a measure of gas density. Thus, in a constant pressure system, the actual pressure must be corrected for temperature by

$$p = p_{\text{actual}} \frac{T_{\text{standard}}}{T} \tag{1}$$

to give a density-equivalent pressure. Although gas density is a more basic quantity for breakdown, pressure and temperature are used because of precedence set by earlier work[2–5] employing the directly measurable quantities. The equation for breakdown applicable to uniform conditions and those nonuniform conditions where E_e/p varies due to electric field gradients cannot be used, in general, in the case where E_e/p varies due to gradients in gas density. In this section, an approximate solution and then an exact solution involving a more general equation will be discussed.

Breakdown theory is based on the equation of electron continuity which equates the various rates of electron production and loss. If the rate of electron production in some region exceeds the rate of loss, we have the possibility of a breakdown occurring. This production process is described for most cases by the following electron continuity equation when no density gradient exists:

$$\frac{\partial n}{\partial t} = \nabla^2 (Dn) + \left(\frac{\nu_n}{D}\right)(Dn) \tag{2}$$

where n is the electron density,

D is the electron diffusion coefficient,

ν_n is the net rate of electron production.

The threshold for breakdown for CW conditions occurs when $\partial n/\partial t = 0$ (*i.e.*, an infinitely long interval of time is allowed for the electron density to build up). The quantity ν_n/D is a function of the breakdown parameter (E_e/p) which, in general, is a function of position because of the spatial variation of electric field and pressure. Therefore, the solution of (2) often requires numerical methods. The determination of the breakdown condition proceeds by solving (2) for eigenfunctions satisfying the boundary conditions and then using the eigenvalues for determining ν_n/D which in turn gives values for E_e/p. In other words, solutions are sought for which $\nabla^2\varphi/\varphi = \text{constant}$ $(\varphi = Dn)$.

The above equation may be used for an approximate but instructive solution. In the vicinity of a hot surface, where E_e/p is essentially constant, there exists a boundary layer of gas across which the temperature drops from its value at the wall down to that of the bulk gas as sketched in Fig. 1(a). Since this system is at constant pressure, the net electron production may be high enough in the hot film to cause breakdown even though there may be no electron production remote from the surface. In the bulk of the gas, the production rate attains negative values in air because of electron attachment to molecules.

In order to evaluate the effect of the film of hot gas, a piecewise constant coefficient solution of (2) was tried as an approximation. A step change in temperature from the wall to the ambient gas temperature was assumed [Fig. 1(b)]. The solution for (Dn) within the films is a sinusoid and, in the bulk of the region, hyperbolic cosine. The boundary conditions require that the function vanish at the walls and that the electron density and electron diffusion current be continuous at the film edge. These requirements yield the following breakdown relationship:

$$\frac{1}{D_0 k_0} \coth k_0 \left(\frac{d}{2} - L\right) = \frac{1}{D_w k_w} \tan k_w L \tag{3}$$

where the subscripts indicate with which region (Fig. 1) the quantities are associated. Eq. (3) is transcendental and can be solved by graphical methods. To complete the solution, an empirical curve for ν_n/D as a function of E_e/p in air[3] was used. With the breakdown criterion of (3), it is useful to normalize the variables to E_e/p_0, $p_0 L$ and T_w/T_0 where p_0 is the density-equivalent pressure of the bulk of the gas.

In terms of these parameters, the solution of (3) is plotted in Fig. 2 for the high pressure case (*i.e.*, the effective field strength equals the rms value of field strength[3]). If the surfaces and the gas are in thermodynamic equilibrium, the breakdown threshold value of E_e/p_0 is 31.5 (v per meter per mm Hg). Since the value of E_e/p_0 in the bulk of the gas is used as a parameter, the breakdown threshold in the worst case will be reduced by the ratio T_0/T_w as indicated by the asymptotes on the right hand portion of Fig. 2. On the other hand, if the value of $p_0 L$ can be reduced to below a value of about 3, the breakdown threshold is essentially unaffected by the presence of the hot surface.

The approximate solution predicts significant increases in breakdown thresholds with small values of $p_0 L$; but, a more exact continuity equation will be solved to substantiate this. The application of the ordinary continuity equation to breakdown near hot surfaces required simplifying the problem by assuming the existence of two uniform regions. In a more exact treatment, because of the spatial variation in ν_n and D for gas density gradients, the ordinary continuity equation (2)

[6] R. M. White and R. H. Stone, "Gaseous breakdown in pressurized microwave components," *Electrnoics*, pp. 45–47; April 20, 1962.

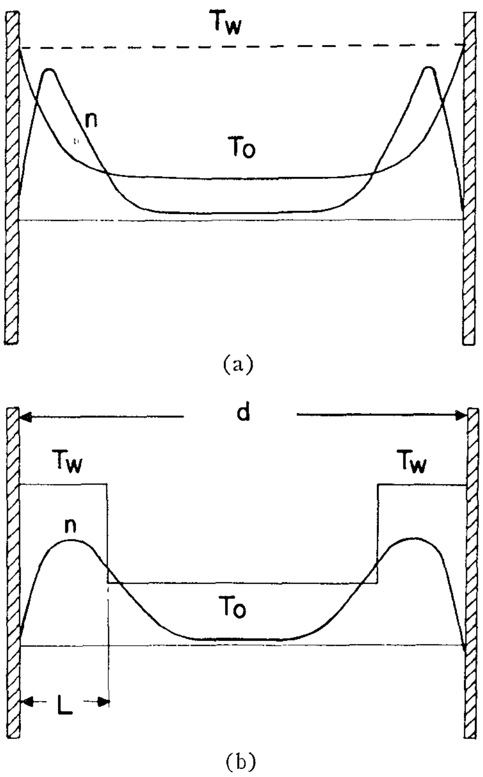

Fig. 1—Physical models used in analyzing breakdown at hot surfaces. (a) Actual model. (b) Approximate model.

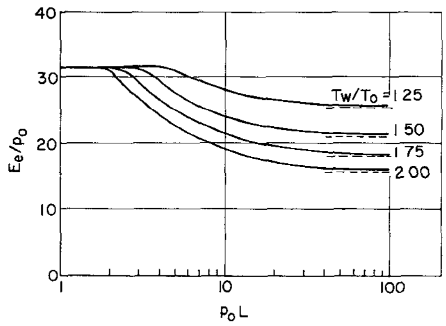

Fig. 2—Normalized solution for breakdown at a hot surface (approximation by piecewise linear solutions).

is modified as indicated in Appendix I, becoming for the one dimensional case

$$\frac{\partial^2}{\partial x^2}\psi + \frac{\nu_n}{D}\psi + \frac{1}{\phi}\frac{\partial \phi}{\partial x}\frac{\partial \psi}{\partial x} = 0 \tag{4}$$

where

$$\psi = \frac{Dn}{\phi}$$

$$\phi = \frac{T}{T_0}.$$

Here T is a function of position and T_0 is the temperature of the bulk gas. The variable x is the distance measured normal to the surface and the quantity ϕ embodies the spatial temperature variation. If ϕ is a constant, this equation reduces to the ordinary breakdown equation. The more complex function ψ containing the electron density n must still satisfy the boundary condition of vanishing at the walls. Since an exact theory of film thickness is not available, an exponential spatial variation of temperature was assumed. The resulting form for the parameter ϕ is

$$\phi = \frac{T}{T_0} = 1 + \left(\frac{T_w}{T_0} - 1\right)e^{-x/L}. \tag{5}$$

The quantity L is defined as the film thickness.

A digital computer solution for the single plate problem was carried out using the more general continuity equation (4). The boundary conditions used were: the function ψ vanishes at the wall and reduces to the hyperbolic form at remote distances. Only the one plate problem was set up for the solution because it was anticipated that the film thickness would be small for solutions of interest. The results of the computer solution for the single plate problems are shown in Fig. 3. The values of p_0L where E_e/p_0 begins to drop are in the same range as predicted by the approximate piecewise linear solution of the two plate problem (Fig. 2.) However, the range of p_0L over which the transition occurs to the limiting case of breakdown controlled by the hot surface is significantly greater. This is due to the extra term in the continuity equation and the more gradual change in temperature in the film.

Numerical solutions for the spatial variation of $\psi = Dn_e/\phi$ were obtained as part of the over-all solution for breakdown. Since both D and ϕ do not vary widely, the function ψ is a fair indication of the electron density distribution during the initial stages of breakdown. The form of the function ψ does not change with time for CW breakdown conditions, but in amplitude. Several cases showing how ψ peaks near the hot surface are given in Fig. 4 using a normalized distance $S = p_0x/p_0L$. The curves clearly illustrate that the electron density becomes more highly localized at the hot surface as the normalized film thickness increases.

The analysis has shown that breakdown near a hot surface is strongly dependent upon the size of the boundary layer of hot gas. Since the film thickness can be reduced by increasing air flow past the surface, it is conceivable that a sufficiently rapid flow would inhibit the breakdown even if the temperature of the hot surface remains unchanged. Appendix II shows that the film thickness for an exponential variation in temperature is

$$L = \frac{3}{N_p^{1/3}}\sqrt{\frac{uy}{v_\infty}} \tag{6}$$

where u is kinematic viscosity (0.150 cm^2/sec for air),

v_∞ is the gas velocity in the main stream,

N_p is the Prandtl number (0.68 for air),

y is the distance from the leading edge of the surface.

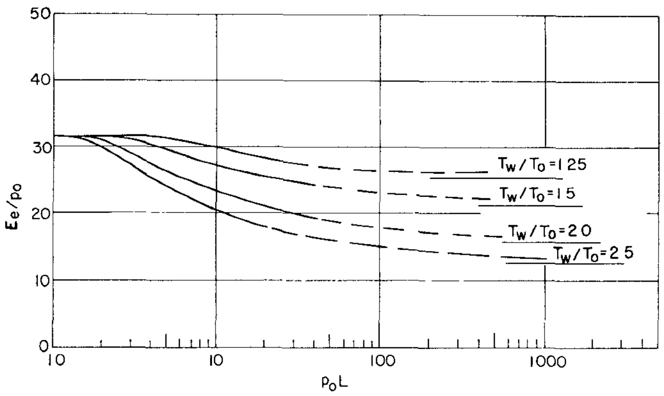

Fig. 3—Normalized solution for breakdown at a hot surface (exact digital computer solution).

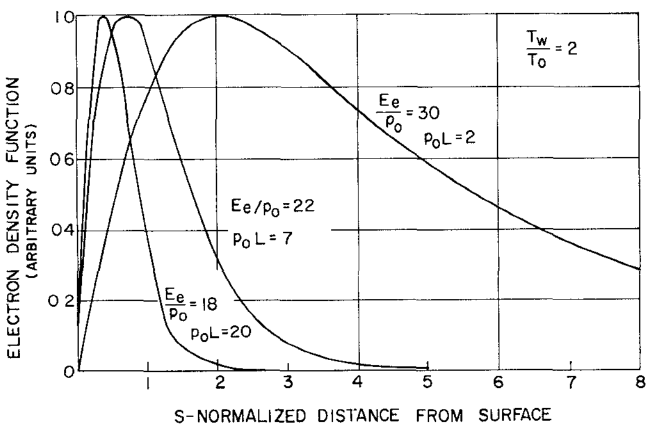

Fig. 4—Variation of ψ, the electron density function, near the hot surface.

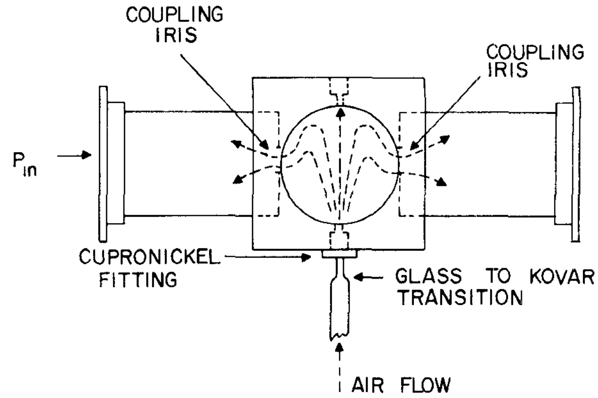

Fig. 5—Sketch of resonant cavity with air flow used for hot surface breakdown study.

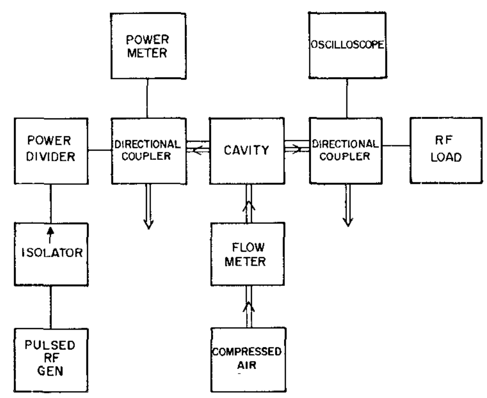

Fig. 6—Block diagram of microwave circuit used for breakdown study.

An order of magnitude calculation shows that this thickness is 0.5 mm for the maximum flow conditions in the experimental work discussed below. For reference, $u=0.150$ cm²/sec for air, $N_p=0.68$ and, in the above calculation, $y=2$ cm and $v_\infty=1240$ cm/sec.

Experimental Results

Experiments were undertaken to evaluate the importance of the heated boundary layer upon breakdown. In general, the breakdown power was measured as a function of the air flow past the hot surface and the temperature ratio between the hot surface and the cooler gas. In the experiments, use was made of a resonant TM_{010} cavity with a narrow height to provide an approximate parallel plate geometry. The cavity was resonant at 5.5 Gc; its height was 0.300 inch and its diameter 1.643 inches. One end of the cavity was removable. The cavity was heated and a stream of pre-cooled air was supplied as illustrated in Fig. 5. The purpose for pre-cooling the air was to partially counteract the heating of the gas as it entered through a small hole in the cavity wall. The microwave circuit arrangement is illustrated in Fig. 6. In taking data, the power was varied (by the power divider) slowly to the point of breakdown which was indicated by the first signs of distortion of the pulse transmitted through the cavity. The cavity was checked before each measurement to make sure that it was operating at its resonant frequency to offset any drifting. The wall temperature of the cavity was monitored with a thermocouple mounted outside the resonant cavity but near the center of one of the flat surfaces. The air was pre-cooled by passing it through a coil in a dry-ice alcohol bath.

Temperature calibration experiments with a gas flow were conducted before the breakdown experiments by placing an additional thermocouple into the center of the cavity to monitor the gas temperature simultaneously with the wall temperature. Fig. 7 shows some of the results of the calibration experiments. Note that the lowest ratio of gas to wall temperature was 0.75 and that the ratio did not continue to decrease with increasing flow rate. A simple preliminary experiment indicated that the percentage change of air pressure in the cavity over that of atmospheric pressure was negligibly small.

The breakdown measurements were taken and interpreted in the following fashion. As the cavity was slowly heated, the change of breakdown power with temperature was monitored; when the temperature finally stabilized, the air flow was introduced and then breakdown power values were measured as a function of flow rates. The wall temperature was also recorded, since it varied with flow rate because of the cooling action of the gas.

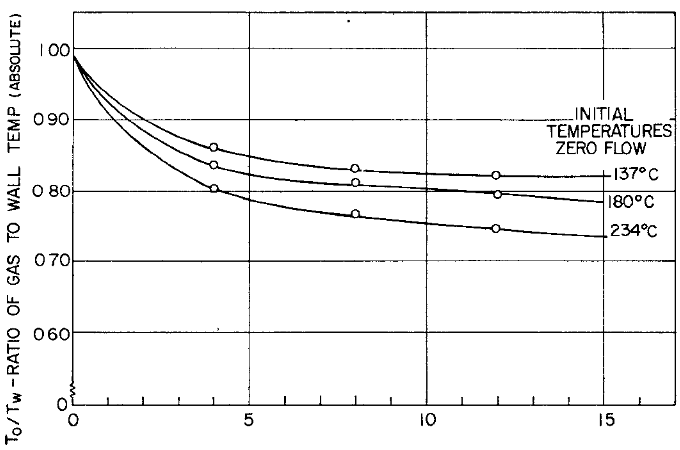

Fig. 7—Temperature calibration of cavity and gas stream as a function of gas flow rate.

To aid in interpreting the measurements in which the wall temperature could not be held fixed, consider

$$\frac{P}{p_w^2} \propto \left(\frac{E}{p_w}\right)^2 = \left(\frac{E_w}{p_w}\right)^2\left(\frac{E}{E_w}\right)^2 \tag{7}$$

where P and E are the breakdown power and field strength, respectively, for a wall temperature T_w; p_w is the density-equivalent pressure corresponding to the gas in equilibrium with the hot wall and E_w is the field strength corresponding to breakdown if the gas were in equilibrium. In general, in the nonequilibrium case, $E > E_w$. Eq. (7) can be normalized

$$\frac{P/p_w^2}{(P/p_w^2)_0} = \frac{(E/E_w)^2}{(E/E_w)_0^2}\,\frac{(E_w/p_w)^2}{(E_w/p_w)_0^2} \tag{8}$$

where the subscript "0" denotes values for a known equilibrium case. But in any equilibrium case, E/p is a constant independent of temperature, at high pressures, and, noting further that in equilibrium $(E/E_w)_0 = 1$ and that $p \propto 1/T$, (8) becomes

$$\frac{PT_w^2}{(PT_w^2)_0} = \left(\frac{E}{E_w}\right)^2. \tag{9}$$

Thus, the value of PT_w^2 in the nonequilibrium case is simply proportional to the square of the ratio of the actual breakdown field strength to that which would have been found if the gas had been in equilibrium with the hot surface. The upper limit to the value of (9) is determined by the value of E corresponding to that for breakdown of the bulk gas (*i.e.*, when the hot film of gas becomes negligibly small). Thus,

$$\left.\frac{PT_w^2}{(PT_w^2)_0}\right]_{\max} = \left(\frac{T_w}{T_0}\right)^2. \tag{10}$$

Experimental results treated in accordance with (10) are given in Figs. 8 and 9 in terms of PT_w^2 as a function of gas flow. Values of PT_w^2 are normalized to a value of unity for zero flow. Two initial wall temperatures are

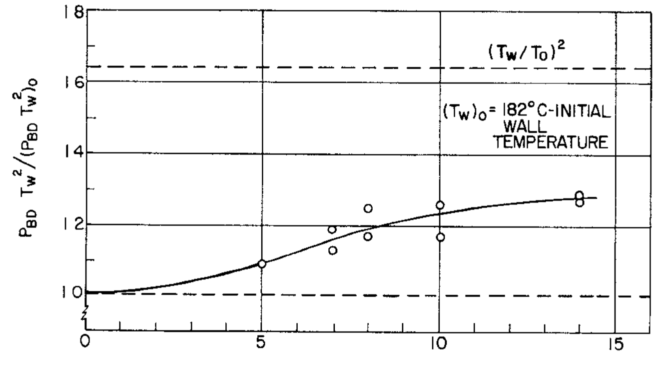

Fig. 8—Normalized breakdown power measurements of a hot surface in the presence of gas flow for an initial wall temperature of 182°C.

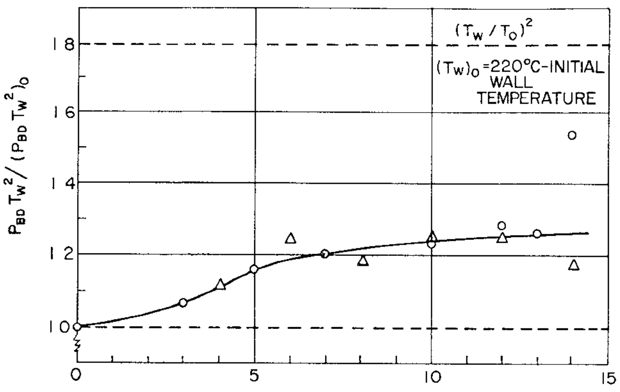

Fig. 9—Normalized breakdown power measurement of a hot surface in the presence of gas flow for an initial wall temperature of 220°C.

shown. The flow rate is in 6.13 cubic feet per hour and 15 corresponds to a velocity of approximately 1200 cm/sec. The results generally exhibit the predicted behavior at medium rates of flow—there is a significant increase above the dashed reference line corresponding to a state of equilibrium between the hot surface and the gas. However, the final levels reached by the experimental value of PT_w^2 fall short of the upper limit calculated using (10). Additional sets of data, not shown, have indicated, in several cases, a downward trend at the high flow rates. Although some scattering of the data is evident and the upper limits were not reached, the experiments do exhibit a significant increase in the quantity PT_w^2 which is too large to be explained by experimental error. Fig. 10 shows that PT_w^2 as a function of temperature is relatively constant for zero flow rate. The small increase might be due to the presence of natural convective currents or possibly due to the cavity Q dropping somewhat at higher temperatures. When the gas was forced through the cavity and there was no temperature difference between the stream and the cavity, it was found that there was no effect on the breakdown threshold. Thus, it is the effect of the flow of cooled air that raises the breakdown threshold and not the mere air movement alone.

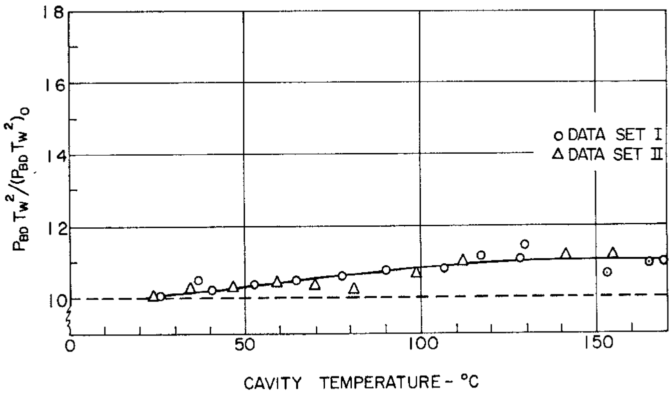

Fig. 10—Normalized breakdown measurements of a hot surface with zero gas flow as a function of wall temperature.

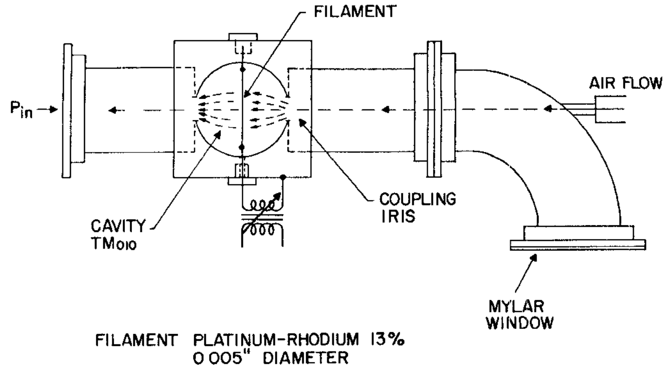

Fig. 11—Sketch of resonant cavity for studying breakdown at a hot wire in the presence of air flow.

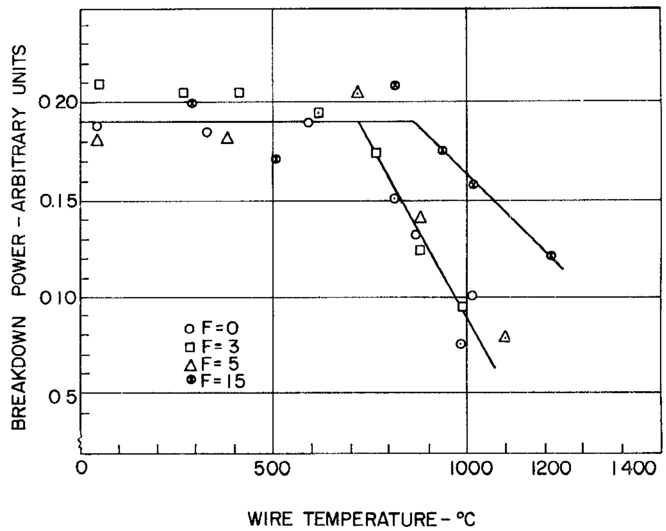

Fig. 12—Results of breakdown measurements with a hot wire in the presence of gas flow.

A second experiment was conducted to obtain qualitative confirmation of the theory with an interesting geometric configuration which unfortunately is difficult to analyze. In this experiment, a wire filament was stretched across the cavity parallel to the two flat faces, transverse to the direction of propagation and passing through the region of maximum E field. The wire tends to increase the local electric fields and thus lower the cavity breakdown threshold. The wire filament was heated by passing current through it as shown in Fig. 11. Thus, the filament not only alters the local fields but it also alters the local gas density as its temperature is varied. The object in this experiment was to determine the extent to which air flow past the wire changes the breakdown threshold.

The microwave circuit arrangement in this experiment is the same as before except that room temperature air is fed in at the bend (Fig. 11), passes through the input iris and across the wire filament. The wire used was platinum-rhodium 5 mils in diameter. The temperature of the filament was monitored by measuring the change of electrical resistance and using values of relative resistance vs temperature found in the American Institute of Physics Handbook. The experimental procedure involved fixing the gas flow rate and changing the current carried by the wire. At each value of current, the resistance and breakdown power (in the presence of a radio active source) were determined.

The resulting measurements of breakdown power vs temperature for a number of gas flow rates are shown in Fig. 12. For several small values of flow rate, the points fall on roughly the same breakdown curve. The important observation is that after a certain temperature has been reached the breakdown threshold begins to fall rapidly with further increases in temperature. At the maximum flow rate (15 units or 1240 cm/sec), the curve breaks at a higher value of temperature and the points are higher than those for small values of flow rate. Again the difference in breakdown between flow rates is interpreted as being due to differences in film thickness—a reduced region of electron production implies greater electron diffusion losses and therefore higher thresholds for breakdown.

Some insight into these results can be gained by reference to the earlier analysis, Fig. 3. The theoretical solutions predict that the reduction in the breakdown threshold begins when p_0L exceed about 1 (mm Hg-cm) for high temperature ratios. Since these experiments are conducted at atmospheric pressure, the film thickness should exceed approximately 1×10^{-3} cm suggesting that the heated wire may be treated as a flat surface. The breakdown measurements in Fig. 12 show that quite high temperatures (700°C) must be reached before the film thickness becomes large enough to reduce the breakdown threshold. Although in these preliminary experiments with a hot wire the values of some of the pertinent variables are not accurately known, the increase in breakdown threshold with gas flow rate is significant as is the threshold in temperature where the breakdown power first begins to decrease.

Conclusions and Discussion

With reference to Fig. 3, the experiments with the hot wire involve the region of p_0L near unity while the hot cavity experiments involve values of p_0L which must be of the order of 100. Using the theory on film thickness, the smallest values of p_0L in the hot cavity are calculated to be 40 and roughly 0.2 for the heated wire. Thus, both ends of the transition region have been explored. For zero flow at the hot wire, natural convection must be effective in cooling the air adjacent to the fine wire suggesting that small foreign particles must reach temperatures in excess of 700°C in order to initiate failures in a waveguide system. From the experimental results, we conclude that the theory developed is useful and leads to additional insight as to what conditions are important for controlling failure mechanisms related to hot surfaces.

This work, on the practical side, demonstrates that air flow increases the breakdown threshold at a hot surface. A factor contributing to this increase is the accompanying reduction in the thickness of the hot film. Another point of interest is that the reduction of the breakdown threshold by a hot surface in a waveguide system can be overcome by providing a gas flow sufficient only to reduce thin film of heated gas, rather than providing the much greater flow required to completely cool the surface.

Appendix I

Derivation of the More General Breakdown Equation

A more general breakdown equation can be derived from the Boltzmann transport theory as treated by W. P. Allis[7] and also discussed by S. C. Brown.[8] From their work, an expression for the electron diffusion current can be written as

$$\Gamma = -\frac{4\pi}{3}\int_0^\infty \frac{v^4}{\nu}\nabla_r f^0 dv \tag{11}$$

where f_0 is the arbitrary equilibrium distribution function of which the spatial gradient is taken and ν is the electron collision frequency. The diffusion current is found by integrating over velocity space.

The spatial variation of ν in (11) leads to the more involved breakdown equation as will be shown now. We will make a substitution for the collision frequency in (11) by defining a reference value ν_0 and the function $\phi(r)$,

$$\frac{1}{\nu} = \frac{\phi(r)}{\nu_0} = \frac{1}{\nu_0}\frac{T}{T_0} \tag{12}$$

such that at some reference point in the volume $\phi(r)$ is equal to unity. Essentially, ϕ represents the variation of the molecular density throughout the region and, in particular, variation as a result of temperature gradients. When (12) is substituted into (11), the factor ϕ and the operator ∇_r can be moved outside the integral sign since they do not involve the velocity. The result can be written as

$$\Gamma = -\phi\nabla_r\left[\frac{1}{\phi}\frac{4\pi}{3}\int_0^\infty \frac{\phi v^4}{\nu_0} f^0 dv\right] \tag{13}$$

where ϕ has been reintroduced inside the brackets without affecting the mathematical operations. Employing the definition for the diffusion coefficient D given by Allis,[7] (12) can be written as

$$\Gamma = -\phi\nabla\left(\frac{Dn}{\phi}\right). \tag{14}$$

The electron continuity equation which expresses the rate of change of electron density in a differential volume is

$$\frac{\partial n}{\partial t} = -\nabla\cdot\Gamma + \nu_n n. \tag{15}$$

With the substitution of (14) into (15), one obtains

$$\frac{\partial n}{\partial t} = \frac{\nu_n}{D}\left(\frac{Dn}{\phi}\right)\phi + \phi\nabla^2\left(\frac{Dn}{\phi}\right) + (\nabla\phi)\cdot\nabla\left(\frac{Dn}{\phi}\right). \tag{16}$$

In (16), it is seen that the spatial variation of the molecular density gives rise to a third term not found in the conventional breakdown equation. In view of the previous work on breakdown, it is apparent that we should define a quantity

$$\psi = \frac{Dn}{\phi} \tag{17}$$

which is the function for which solutions are sought. The dominant quantity, however, is still n, the electron density.

Appendix II

Film Thickness

An expression for film thickness at a hot surface can be derived by considering that the velocity of the stream is zero at the surface and rises to the free stream velocity some distance from the surface. This velocity variation arises from the viscous forces. Hence, the energy carried away from the surface is transmitted to the gas immediately adjacent to the surface by conduction and this energy must also equal the heat carried away by convection in the film. Thus,

$$-k\left(\frac{\partial T}{\partial x}\right)_w A = h(T_w - T_0)A \tag{18}$$

where k is the thermal conductivity of air, h is the convective heat transfer coefficient and the subscript w denotes evaluation at the hot surface. From (18), the ratio k/h is

[7] W. P. Allis, "Motion of ions and electrons," in "Handbuch der Physik," vol. XXI, Springer-Verlag, Berlin, Germany; 1956.

[8] S. C. Brown, "High frequency gas discharge breakdown," in "Handbuch der Physik," vol. XXII, Springer-Verlag, Berlin, Germany; 1956.

$$\frac{k}{h} = \frac{T_w - T_0}{(\partial T/\partial x)_w}. \qquad (19)$$

Substituting an exponential form for T (5) into (19), the result is

$$L = k/h \qquad (20)$$

where L is the exponential decrement and defined as the film thickness.

This discussion is complicated by the fact that h is a function of the distance along the hot surface in the direction of gas flow. A simple theoretical result that can be obtained is[1]

$$h_y = 0.332 N_p^{1/3} k \sqrt{\frac{v_\infty}{uy}} \qquad (21)$$

where y is distance measured from the leading edge along the hot surface, the quantity N_p is the dimensionless Prandtl number of the gas, u is the kinematic viscosity term and v_∞ is the free stream velocity. From (20) and (21), we obtain an equation for L,

$$L = \frac{3}{N_p^{1/3}} \sqrt{\frac{uy}{v_\infty}}. \qquad (22)$$

ACKNOWLEDGMENT

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The Application of the Focussed Fabry-Perot Resonator to Plasma Diagnostics*

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Summary—**The use of a focussed Fabry-Perot resonator at microwave frequencies for plasma diagnostics is discussed. It is shown theoretically that improvements in sensitivity in the measurements of the properties of transparent plasmas and dielectrics of two to three orders in magnitude can be expected. Losses have been neglected. It is indicated that under certain circumstances, refractive index changes in the gaseous environment may be significant. A method of measuring these changes is included. The extension of these techniques to the optical part of the spectrum is discussed and is shown to be promising. Experimental results, obtained with the use of a cavity at 70 Gc, are presented and appear to confirm the main predicted features.**

INTRODUCTION

CONVENTIONAL, high-frequency, focussed probes have been successfully used to measure the ionization in wakes behind projectiles in flight at hypersonic velocities, where the wake diameters are comparable to the wavelength.[1] For small diameter wakes, short wavelengths are essential and the resulting lower limit to the minimum measurable electron density at these wavelengths may be too high for some applications. The use of lower frequencies would produce measurable effects at lower densities, but the degradation of resolution might offset this advantage. Since a rigorous solution of the scattering problem (one which would allow for the finite sizes of the beam and wake) is not always practical, there is a need for a technique which retains the resolution of the shorter wavelength probes but provides several orders of magnitude improvement in sensitivity. A system that may provide these features is described.

The basic limitation in sensitivity of a focussed microwave probe lies in the fact that the beam is transmitted only once through the plasma of interest. For a fixed frequency, the limit is reached when the plasma properties are such that the phase shift of the transmitted wave is reduced to the minimum measurable level. This is illustrated in the idealized case of normal incidence of a uniform plane wave on a uniform plasma slab.

For simplicity, the electron collision frequency is assumed to be zero and the plasma to be underdense; that is, $(\omega_p/\omega)^2 \ll 1$ where

ω is the operating frequency
ω_p is the plasma frequency of the ionized medium.

The phase shift of the transmission coefficient caused by the introduction of the slab in the beam can be shown to be[2]

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